

# Evaluating the Data Integrity of Memory Systems by Configurable Markov Models

M. Ottavi, L. Schiano, F. Lombardi  
Department of Electrical and Computer Engineering  
Northeastern Univ. Boston, USA  
{mottavi, lschiano, lombardi}@ece.neu.edu

S. Pontarelli, G.C. Cardarilli  
Department of Electronic Engineering  
University of Rome, Italy  
{pontarelli, cardarilli}@ing.uniroma2.it

## Abstract

*In this paper, a novel method for the evaluation of the Bit Error Rate (BER) as measure for assessing data integrity in memory systems is proposed; such method improves modeling by introducing configurability features in the Markov chains to account for environmental and operational changes. For modeling erasures and random errors, the occurrence of new time-varying features is introduced in the analysis to characterize the behavior of memory systems for space applications (using Reed-Solomon codes as EDAC). Moreover, differently from existing techniques, the nature of these features (such as scrubbing and the effects of the so-called South Atlantic Anomaly on SEU rates) is assessed using a deterministic framework.*

## 1 Summary and Conclusions

On-board data-collection instruments of satellites usually generate a large volume of data; due to tight avionic requirements (such as mission time) this data must be stored in high reliable on-board VLSI memories.

The objective of this paper is to evaluate data integrity of a VLSI memory system affected by transient and permanent faults when EDAC (Error Detection and Correction) codes [2], sparing and scrubbing [6] are used. BER (bit error rate) is used as figure of merit for assessing data integrity in presence of erasures and random errors in avionic/space applications. Differently from previous techniques, a novel Markovian model is proposed; this is based on so-called configurable Markov chain by which variations in design parameters and environment conditions are accounted. This allows enhanced versatility in the modeling process with no substantial increase in its complexity. The proposed technique is applied under different scenarios to memory system with Reed-Solomon codes as EDAC. In a first example, a memory system using scrubbing is considered. Differently from previous work [3], scrubbing is here realistically modeled as a deterministic operation. In a second example, the effects of the South Atlantic Anomaly on the operation of memories in satellites is investigated by considering bursts of SEU (single event upsets). The results presented in this paper substantiate the need to accurately incorporate these new features into the evaluation of BER. The paper is organized as follows: The Markov model of a generic  $(n, k)$  Reed-Solomon code is presented in Section 2. Section 3 presents the basic framework of configurable Markov models. Section 4 illustrates how the presented method can be used to evaluate scrubbing operations in a VLSI memory system. Results are compared with those obtained with a probabilistic model. In Section 5 the South Atlantic Anomaly effects are modeled to estimate the BER in the presence of bursts of high energy particles.

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## 2 Reed-Solomon codes

Different EDAC codes have been proposed for memory systems in avionic applications. Over the last few years, Reed-Solomon codes have been widely advocated due to their advantageous features for EDAC as well as VLSI implementation. [3] has proposed a Markov model to analyze Reed-Solomon (RS) coded memory systems with scrubbing. Without loss of generality, this model is used as basis for this paper. A  $n$ -states Markov model leads to a set of  $n$ -coupled differential equations in  $n$  variables which represent the states modeling the system behavior:  $P'(t) = \mathbf{A}P(t)$  where  $\mathbf{A}$  is the transition matrix. The solution of the Markov model is a state probability vector  $P(t) = [P_{S(0)}(t), P_{S(1)}(t), \dots, P_{S(n)}(t)]$  (where  $P_{S(i)}(t)$  is the probability to be in state  $S(i)$  at time  $t$ ). In the model of [3] the probability  $P_{S(n)}(t)$  represents the probability that the system provides an erroneous codeword. The percentage of erroneous codewords read over a certain time  $t$  (i.e.

the BER) is defined as

$$BER(t) = m \cdot \frac{(n-k)}{k} \cdot P_{S(n)}(t)$$

where  $m$  is the number of bits per symbol. When a codeword is uncorrectable, the number of erroneous bits is assumed to be equal to the number of bits in  $n-k$  symbols, i.e. the Hamming distance between two codewords.

### 3 Configurable Markov Models

Differently from conventional approaches, the proposed technique relies on configurability to model deterministic events such as scrubbing. Furthermore, it introduces run-time varying values for the transition rates, thus permitting to model time dependent burst effects (such as encountered in avionics systems for the increase in high energy particles due to the pass of a Low Earth Orbit satellite through the South Atlantic Anomaly). Initially a mathematical model is presented; consider a generic Markov Chain representing the time-dependent evolution of a memory system in which configurability takes place to account for changes (also known as perturbations). The set of differential equations representing the system is given by:  $P(t + \Delta t) = \mathbf{B}P(t)$ ; the matrix  $\mathbf{B}$  is related to the transition matrix  $\mathbf{A}$  as  $\mathbf{B} = \mathbf{A} \cdot \Delta t + \mathbf{I}$  where  $\mathbf{I}$  is the identity matrix. This relation can be exploited to introduce new features in the evaluation of the memory system provided  $\Delta t$  is sufficiently small. In this case, the time duration of the perturbation (not necessarily its effects) is not significant compared with the overall operational time period of the system. Two different configuration techniques are hereafter considered as examples of a wide set of realistic applications in which configurable Markov models can be utilized. (1) The first technique is based on changing the vector  $P(\bar{t})$  (which represents the state of the system at a given time  $\bar{t}$ ) to a new value given by  $\bar{P}$ . This configuration is suitable for modeling the occurrence of deterministic events. As an example, in Section 4 it is used to model the deterministic nature of the scrubbing operation. (2) The second technique is performed by modifying the value of the matrix  $\mathbf{B}$  for a given period of time  $\bar{T}$ . This approach is useful to model variations in the operating conditions of the memory system. As an example, in Section 5 variations of the SEU rate related to environmental conditions are considered. In both cases, a memory system with RS codes as EDAC and its Markovian model presented in [3] are used in the analysis.

### 4 Deterministic Scrubbing

A scrubbing operation can be modeled deterministically as well as probabilistically [6]. The difference between the

deterministic and probabilistic models for scrubbing can be outlined by considering a simple Markov chain made of only two states. State 1 is the Good state, while following the occurrence of an error the system moves to state 2 (the Fail state). Consider first the probabilistic model. In this case, the occurrence of an error is modeled by a transition with fixed rate  $\lambda$ , while scrubbing is modeled with a transition with rate  $\mu = \frac{1}{T_{sc}}$ . The solution of the probabilistic model is given by

$$P_1(t) = \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t} + \frac{\mu}{\lambda + \mu} \quad (1)$$

$$P_2(t) = 1 - P_1(t) = -\frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t} + \frac{\lambda}{\lambda + \mu} \quad (2)$$

Consider next a deterministic model for scrubbing; in this case, the rate  $\mu$  is not required, while the model of the scrubbing operation must satisfy the boundary conditions of being in state 1 at every scrubbing time.

Therefore solution is now given by

$$P_1(t) = e^{-\lambda(t - \lfloor \frac{t}{T_{sc}} \rfloor T_{sc})} \quad (3)$$

$$P_2(t) = 1 - P_1(t) = 1 - e^{-\lambda(t - \lfloor \frac{t}{T_{sc}} \rfloor T_{sc})} \quad (4)$$

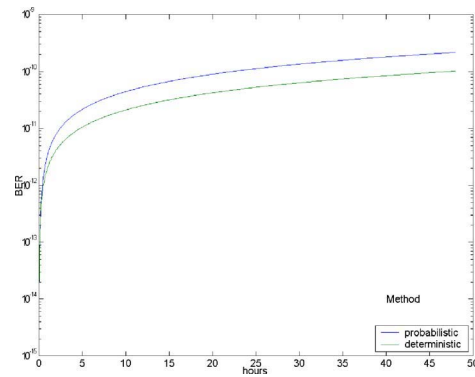
where  $\lfloor x \rfloor$  is the floor function, which gives the largest integer smaller or equal to  $x$ .

The above two solutions yield very different results due to the characterization of the scrubbing operation. While a probabilistic technique is convenient for mathematically modeling, in practice scrubbing has a deterministic execution; due to the stringent requirements for avionic applications, its periodic execution requires a more detailed analysis. Therefore, a comparison of the deterministic and probabilistic models for BER evaluation can be very relevant to scrubbing in a memory system with EDAC codes (in this case a generic RS( $n, k$ ) code is used). Similar boundary conditions apply to the model of [3] if scrubbing is modeled as a deterministic operation. In this case, the probability of a random error (erasure) is put to as 0 (unchanged) while scrubbing takes place. The evaluation of the two models has been performed by using the following parameters:  $T_{max} = 48h$  (maximum time of storage of data i.e. time between two downloads to earth),  $\lambda = 7.3 \cdot 10^{-7}$  (minimum SEU rate),  $\lambda_e = 0$  (no permanent errors), and  $T_{sc} = 900sec$ . Results are shown in Figure 1. By comparing the two models (ignoring permanent faults and using the minimum SEU rate), the difference in percentage is as high as 50%, thus reflecting possible inaccuracies in previous evaluation methods of BER.

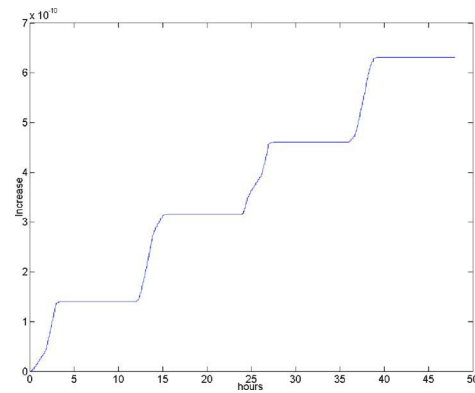
### 5 The South Atlantic Anomaly

In this section, the so-called South Atlantic Anomaly is considered as a space environment for a VLSI memory sys-

tem. It has been observed that above South America starting from about 200 - 300 kilometers off the coast of Brazil, and extending over much of the continent, the Van Allen Belt forms the so-called South Atlantic Anomaly (SAA). A Low Earth Orbit (LEO) satellite passing through this region of space loses the Van Allen radiation belt's protection and is exposed to high energy protons. These hits on VLSI systems have been widely studied in literature [1] [5] [4]. They are cause of many negative temporary as well as permanent effects (which can be viewed as an accelerated aging of the electronic devices). SAA presents a grave threat to VLSI systems in LEO orbits tilted more than 45 degrees on the Earth's equator; even the exposure of few minutes to a high particle flux can be disastrous for the operation of a satellite. Given the orbit parameters of a LEO satellite it is then possible to calculate the number of orbits affected by the SAA (denoted by  $n$ ) between two consecutive data downloads to the earth station as well as the duration of the exposure for each orbit (hereafter denoted to as  $T_{pass}$ ). These values can then be used as input to a configurable Markov model for computing the expected SEU rate of a given code and scrubbing period. From Kepler's equations, a satellite with altitude  $h$  on the ground has an orbital period  $T = \frac{2\pi}{R} \sqrt{\frac{(h+R)^3}{g}}$ , where  $R$  is the earth radius and  $g$  is the gravitational acceleration.  $T_{pass}$  is computed as the product of the ratio of the SAA orbital length in radians (i.e.  $\pi/4$ ) over the entire orbital (i.e.  $2\pi$ ) and the orbital period  $T$ , i.e.  $T_{pass} = \frac{\pi/4}{2\pi} T = T/8$ .  $n$  is effectively the ratio between the maximum latitude by which the SAA spans (i.e.  $\pi/2$ ) and the angle by which the earth rotates when the satellite completes one orbit (i.e.  $2\pi \frac{T}{24h}$ ); hence,  $n = \frac{\pi/2}{2\pi \frac{T}{24h}} = 6h/T$ . In a 24 hour period depending on the orbital parameters, the satellite passes approximatively on the same trace on the ground. So, the actual number of orbits affected by the SAA is  $2n$ , i.e. the sum of the  $n$  orbits for which the satellite travels the SAA from north to south and the  $n$  orbits from south to north (two passes). Having established these parameters a configurable Markov chain has been used to calculate the SEU rate of the considered memory system for a time-dependent  $\lambda$ . BER versus time has been calculated for a RS(18,16) memory with no scrubbing and permanent faults. Bursts of  $\lambda = 5 \times 10^{-5}$  have been considered for the SAA region, while  $\lambda = 7.3 \times 10^{-7}$  has been adopted during the remainder of the satellite's orbital period. A comparison between the BER plot obtained by considering the SAA and the one obtained by only considering a smaller SEU rate is reported in Figure 2 in which the BER increase shows an expected step-wise relation with time. The effect of the SEU rate increase is significant and appropriate corrective actions (such as possibly a different RS code) can be assessed based on this plot.



**Figure 1. Comparison between deterministic and probabilistic models for scrubbing**



**Figure 2. BER increase to account for the SAA**

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